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Foundations of Sobolev Spaces and Applications to Elliptic Partial Differential Equations

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Abstract

We develop key notions concerning Sobolev spaces, including approximation theorems (which ensure the density of smooth functions), extension theorems (allowing functions to be extended beyond their original domain), and the Trace Theorem, which associates to each function in a Sobolev space a well-defined boundary value. This theorem plays a central role in understanding boundary conditions in the weak setting. In particular, the characterization of functions with zero trace allows us to interpret the standard Sobolev space as the space of Sobolev functions that vanish on the boundary. The Hilbert space structure allows the application of powerful tools from functional analysis, in particular, the Lax-Milgram Theorem, whose hypotheses are verified through the Poincaré inequality (ensuring coercivity) and continuity estimates. The Lax-Milgram theorem is used to establish the existence and uniqueness of weak solutions to second-order linear uniformly elliptic partial differential equations. This theoretical framework provides a rigorous and unified approach to solving such equations in the weak sense.

References

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