VII International Meeting in Mathematical Sciences
Huaraz, Perú
December 15–17, 2025

Analysis of an Unfitted HDG Method for a Class of Non-linear Elliptic Problems using Local Proximity Conditions

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Abstract

We study Hibridizable Discontinuous Galerkin (HDG) discretizations for a class of non- linear interior elliptic boundary value problems posed in curved domains where both the source term and the diffusion coefficient are non-linear. We consider the cases where the non-linear diffusion coefficient depends on the solution and on the gradient of the solution. To sidestep the need for curved elements, the discrete solution is computed on a polygonal subdomain that is not assumed to interpolate the true boundary, giving rise to an unfitted computational mesh. We show that, under mild assumptions on the source term and the computational domain, the discrete systems are well posed. Furthermore, we provide a priori error estimates showing that the discrete solution will have optimal order of convergence as long as the distance between the curved boundary and the computational boundary remains of the same order of magnitude as the mesh parameter. achieved through a local proximity condition. This condition limits the minimum size that elements near the boundary of an admissible triangulation can attain.

References

[1] Cockburn, B., Solano, M., *Solving Dirichlet boundary-value problems on curved domains by extensions from subdomains.*, SIAM J. Sci. Comput. 34(1), A497–A519 (2012)

[2] Sánchez, N., Sánchez-Vizuet, T., Solano, M.E., *A Hybridizable discontinuous Galerkin solver for the Grad-Shafranov equation.*, Comput. Phys. Commun. 235, 120–132 (2019)

[3] Sánchez, N., Sánchez-Vizuet, T., Solano, M.E., *A priori and a posteriori error analysis of an unfitted HDG method for semi-linear elliptic problems.*, Numer. Math. 148(4), 919–958 (2021).

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