

Algunos índices de foliaciones y aplicaciones

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Abstract

Presentaremos una relación entre el número de Milnor, el número- x y el número de Tjurina de una foliación con respecto a un divisor balanceado efectivo de separatrices. Además, usando el índice de Gómez-Mont-Seade-Verjovsky de una foliación mostraremos la relación entre la multiplicidad y el número de Tjurina con respecto a curva reducida singular. Estos resultados locales tienen consecuencias globales, daremos una cota superior e inferior para el número de Tjurina global de una foliación en el espacio proyectivo complejo. Este trabajo es en colaboración con Evelia R. García Barroso y Arturo Fernández Pérez.

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